NEET-UG — AUDITED SIMPLE EDITION

Gravitation

Physics by Rajeev Agarwal

Study sequence: Force → gravity and its variation → potential and energy → escape → Kepler's laws → satellites.

Symbols you need

M, R	Mass and radius of the attracting planet.
m	Mass of the small body or satellite.
r	Distance from the planet's **centre**.
h, d	Height above surface; depth below surface.
g_0	Gravity at the surface, ignoring rotation.
V, U	Gravitational potential; gravitational potential energy.
K, E, T	Kinetic energy; total mechanical energy; orbital period.

1. Universal law of gravitation

$$F = \frac{GMm}{r^2}$$

Gravitation is always attractive. For two non-overlapping bodies with spherically symmetric mass distributions, use the distance between their centres. For point masses, use their separation.

$$G \approx 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}, \qquad [G] = \mathsf{M}^{-1}\mathsf{L}^{3}\mathsf{T}^{-2}.$$

If the masses become p and q times their original values and separation becomes k times its original value:

$$\frac{F_{\text{new}}}{F_{\text{old}}} = \frac{pq}{k^2}.$$

When more than one body attracts a particle

Add the forces with their directions. Same direction: add. Opposite directions: subtract magnitudes. Perpendicular forces: $F_{\text{net}} = \sqrt{F_1^2 + F_2^2}$. For any angle θ between them:

$$F_{\text{net}} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}.$$

Use with care. G is universal; g varies with location and planet. Unless stated otherwise, use spherical planets, negligible air resistance, and a satellite much lighter than its planet.

Added formula | Vector form of the gravitational field

$$\vec{g}(r) = -\frac{GM}{r^2}\,\hat{\mathbf{r}} \quad \left(\vec{F} = m\vec{g}\right)$$

For a point source or outside a spherical planet. The unit vector $\hat{\mathbf{r}}$ points outward; the minus sign means gravitational acceleration points inward.

01 — AFTER GRAVITATIONAL FORCE

Acceleration due to gravity

2. Gravity and weight

$$g = \frac{F}{m} = \frac{GM}{r^2}, \qquad g_0 = \frac{GM}{R^2}, \qquad W = mg.$$

These inverse-square formulas apply outside a spherical planet. The SI unit of g is m s^{-2} (also N kg^{-1}). Weight is measured in newtons; mass, in kilograms, does not change with location.

Gravity in terms of average density

Here $\bar{\rho}$ is average density, in kg m^{-3}.

$$M = \frac{4}{3}\pi R^3 \bar{\rho}, \qquad g_0 = \frac{4}{3}\pi G \bar{\rho} R.$$

$$\frac{g_1}{g_2} = \frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{\bar{\rho}_1 R_1}{\bar{\rho}_2 R_2}.$$

At the same average density: $g_0 \propto R$. At the same mass: $g_0 \propto 1/R^2$.

3. At height h above the surface

$$r = R + h, \qquad g_h = g_0\left(\frac{R}{R+h}\right)^2.$$

For small heights only, $h \ll R$:

$$g_h \simeq g_0\left(1 - \frac{2h}{R}\right).$$

Quick check: at $h = R$, distance from centre is $2R$, so $g_h = g_0/4$.

4. At depth d below the surface

For a planet of **uniform density**:

$$r = R - d, \qquad g_d = g_0\left(1 - \frac{d}{R}\right).$$

At $d = R/2$, $g_d = g_0/2$; at the centre, $g = 0$. If height and depth give the same gravity:

$$\left(\frac{R}{R+h}\right)^2 = 1 - \frac{d}{R}.$$

For small h, d compared with R: $d \simeq 2h$.

Use with care. The depth law requires uniform density; the small-height formula requires $h \ll R$. Use the exact height formula whenever the height is a substantial fraction of R.

02 — BEFORE ESCAPE AND SATELLITES

Potential, energy and work

5. Potential and potential energy

For a point mass, or outside a spherical planet, taking zero at infinity:

$$V = -\frac{GM}{r}, \qquad U = mV = -\frac{GMm}{r}.$$

	Potential V	Potential energy U
Meaning	Energy per unit test mass	Energy for the given mass m
SI unit	J kg^{-1}	J
At infinity	0	0

As a body moves outward, V and U increase: they become less negative. Gravity becomes weaker.

6. Change in potential energy

Between two exterior positions:

$$\Delta U = U_f - U_i = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right).$$

$$W_{\text{gravity}} = -\Delta U.$$

If gravity and the external agent are the only forces doing work, slow lifting with no change in kinetic energy gives $W_{\text{ext}} = \Delta U$. Surface to height h:

$$\Delta U = \frac{GMmh}{R(R+h)} = mg_0h\frac{R}{R+h}.$$

For $h \ll R$, $\Delta U \simeq mg_0h$. This familiar formula is an energy **difference**.

Potential from several masses

For point sources, or a point outside each spherical source:

$$V = -G\left(\frac{M_1}{r_1} + \frac{M_2}{r_2} + \cdots\right).$$

Add potentials with their signs; no direction is attached to potential. For the energy of a system of point masses, add $-Gm_im_j/r_{ij}$ for each distinct pair, counting each pair once. **Example:** three equal masses m at the corners of an equilateral triangle of side s have $U = -3Gm^2/s$.

Zero net force between two fixed masses

For two point masses M_1, M_2 separated by D, the zero-force point is between them, at distance

$$x = \frac{D\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}$$

from M_1. It is closer to the smaller mass.

Use with care. Zero net force does not imply zero potential. Also, $U = 0$ at infinity is a chosen reference; $U = mg_0h$ is not the absolute energy relative to infinity.

03 — STANDARD CASES AND GRAPHS

Shells and solid spheres

7. Uniform thin spherical shell: mass M, radius R

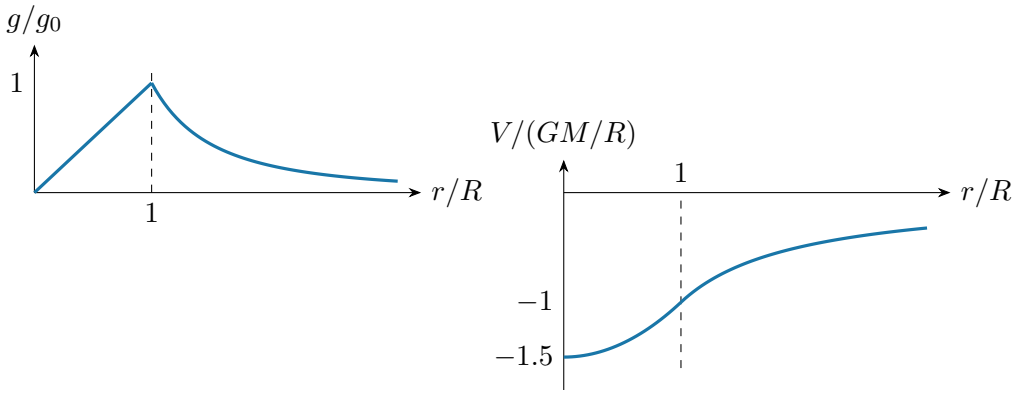
Position	Gravity magnitude g	Potential V
Inside $(r < R)$	0	$-GM/R$ (constant)
Outside $(r > R)$	GM/r^2	$-GM/r$

Inside a shell: no net gravitational force, but the potential is non-zero.

8. Uniform solid sphere: mass M, radius R

Position	Gravity magnitude g	Potential V
Centre	0	$-3GM/(2R)$
Inside $(0 < r < R)$	$g_0 r/R$	$-\dfrac{GM}{2R}\left(3-\dfrac{r^2}{R^2}\right)$
Surface	$g_0 = GM/R^2$	$-GM/R$
Outside $(r > R)$	GM/r^2	$-GM/r$

Recognise the shapes: uniform solid sphere



- In the solid sphere, g rises linearly from zero to its maximum at the surface, then decreases as $1/r^2$.
- Potential is most negative at the centre and approaches zero outside as r increases.
- For a thin shell, the interior g graph is flat at zero and the interior V graph is flat at $-GM/R$; the exterior shapes are the same as above.

Use with care. The solid-sphere interior formulas require uniform density. At an ideal thin shell, use the inside and outside field limits separately; its potential is continuous at the surface.

Added formulas | Field, potential and spherical mass

Field–potential relation

$$g_r = -\frac{dV}{dr}, \quad \vec{g} = -\nabla V$$

Potential difference (exterior)

$$\Delta V = GM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

Extension: arbitrary spherical density $\quad g(r) = \dfrac{G\,M_{\text{enc}}(r)}{r^2}$

g_r is the signed radial field component; $M_{\text{enc}}(r)$ is mass enclosed inside radius r. The potential-difference formula assumes one point mass or exterior spherical field.

04 — USING CONSERVATION OF ENERGY

Escape speed and vertical motion

9. Conservation of mechanical energy

Total mechanical energy is $E = K + U$. With only gravity doing work, it stays constant. Outside the planet, if u is initial speed at r_i and v is final speed at r_f:

$$\frac{1}{2}mu^2 - \frac{GMm}{r_i} = \frac{1}{2}mv^2 - \frac{GMm}{r_f}.$$

$$v^2 = u^2 + 2GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right).$$

If released from rest at r_i, put $u = 0$. For release from rest at infinity, $v = \sqrt{2GM/r}$.

10. Escape speed

Minimum initial speed to reach infinity with zero final speed, ignoring air resistance, planetary rotation and other bodies:

$$v_e = \sqrt{\frac{2GM}{r}} \quad (r \geq R), \qquad v_e(\text{surface}) = \sqrt{\frac{2GM}{R}} = \sqrt{2g_0R}.$$

From rest at radius r, minimum kinetic energy required is GMm/r. Escape speed does not depend on the escaping body's mass. At height h:

$$v_e(h) = v_e(\text{surface})\sqrt{\frac{R}{R+h}}.$$

In terms of the planet's average density:

$$v_e(\text{surface}) = R\sqrt{\frac{8\pi G\bar{\rho}}{3}}, \qquad \frac{v_{e1}}{v_{e2}} = \sqrt{\frac{M_1R_2}{M_2R_1}}.$$

For Earth, surface escape speed is about 11.2 km s^{-1}.

11. Maximum height of a vertical launch

From the surface, for $u < v_e$:

$$H = \frac{Ru^2}{v_e^2 - u^2}, \qquad u^2 = \frac{2g_0RH}{R+H}.$$

For $H \ll R$, $u^2 \simeq 2g_0H$. If $u \geq v_e$, there is no finite maximum height in this ideal radial model. If $u \geq v_e$, speed far away is $v_\infty = \sqrt{u^2 - v_e^2}$.

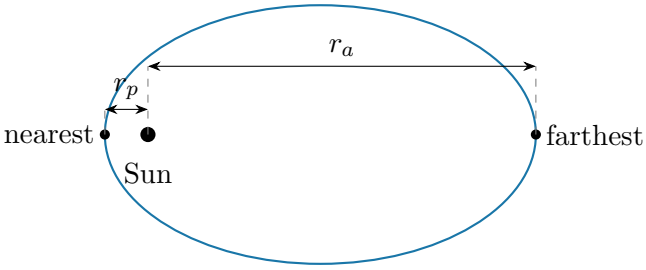
Use with care. In this page's surface-launch formulas, v_e means surface escape speed. Use radial outward motion for the maximum-height result, and a collision-free trajectory for escape.

05 — PLANETARY MOTION

Kepler's three laws

12. First law: the orbit

A planet moves in an ellipse with the Sun at one focus. A circular orbit is a special case of an ellipse.



r_p and r_a are measured from the Sun, not from the ellipse centre. For an ellipse:

$$a = \frac{r_p + r_a}{2},$$

where a is the semi-major axis. For a circle, $a = r$.

13. Second law: equal areas in equal times

The Sun-to-planet line sweeps equal areas in equal time intervals. The planet moves faster when nearer the Sun and slower when farther away. Angular momentum about the Sun is conserved. At the nearest and farthest points, velocity is perpendicular to the Sun-to-planet line:

$$r_p v_p = r_a v_a, \qquad \frac{v_p}{v_a} = \frac{r_a}{r_p}.$$

At other points, it is $rv_\perp$ that stays constant; $v_\perp$ is the component perpendicular to the radius.

14. Third law: period and orbit size

For planets or small satellites orbiting the **same central mass**:

$$T^2 \propto a^3, \qquad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3.$$

$$T = 2\pi\sqrt{\frac{a^3}{GM}}.$$

For circular orbits, replace a by r.

Use with care. Use semi-major axis for an elliptical orbit, not its instantaneous distance from the Sun. The period ratio above assumes the same central mass and negligible orbiting masses.

Added formulas | Kepler's laws in quantitative form

Second law: areal velocity

$$\frac{dA}{dt} = \frac{L}{2m} = \frac{rv_\perp}{2}$$

Third law: explicit constant

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$

L is angular momentum about the attracting mass, a is semi-major axis; the period formula assumes the orbiting mass is negligible compared with M.

06 — AFTER KEPLER'S LAWS

Circular satellites: speed and energy

15. Orbital speed and period

For a small satellite in a circular orbit of radius $r = R + h$:

$$\frac{GMm}{r^2} = \frac{mv_o^2}{r}, \qquad v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{g_0R^2}{R+h}}.$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}, \qquad \omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}.$$

Gravity itself supplies the required centripetal force. The satellite's mass cancels out of v_o and T.

16. Kinetic, potential and total energy

$$K = \frac{GMm}{2r}, \qquad U = -\frac{GMm}{r}, \qquad E = -\frac{GMm}{2r}.$$

$$U = -2K, \qquad E = -K = \frac{U}{2}.$$

Total energy is negative for a bound orbit with zero potential at infinity. The minimum extra energy needed to escape from this circular orbit is

$$E_{\text{extra}} = \frac{GMm}{2r} = |E|.$$

17. Useful connections

$$v_e(r) = \sqrt{2}\, v_o(r), \qquad L = mrv_o = m\sqrt{GMr}.$$

L is the satellite's angular momentum about the planet's centre. For a hypothetical orbit very close to the surface:

$$v_o \simeq \sqrt{g_0R}, \qquad T \simeq 2\pi\sqrt{\frac{R}{g_0}} = \sqrt{\frac{3\pi}{G\bar{\rho}}}.$$

For Earth: $v_o \approx 7.9$ km s^{-1} and $T \approx 85$ min. A real orbit requires enough altitude to avoid the ground and significant atmospheric drag.

If an elliptical satellite orbit is explicitly given

Its total energy is $E = -GMm/(2a)$, where a is the semi-major axis. Do not use the instantaneous r in place of a.

Use with care. $U = -2K$ is an instantaneous relation for a circular orbit. Do not apply it at every point of an elliptical orbit.

Added formula | Another useful circular-orbit identity

$$v_o = \frac{2\pi r}{T} = \sqrt{g(r)\, r}$$

Circular orbit only; r is the radius from the planet's centre, not the altitude. Here $g(r) = GM/r^2$ is the field magnitude at the orbit.

07 — SATELLITE APPLICATIONS

Orbit changes, geostationary and weightlessness

18. Between two circular orbits

For the same satellite moving from radius r_1 to r_2:

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

For a higher circular orbit: **speed decreases, period increases, and total energy increases** (becomes less negative). Energy must be supplied overall. From rest at a non-rotating planet's surface to a circular orbit of radius r:

$$\Delta E = GMm\left(\frac{1}{R} - \frac{1}{2r}\right).$$

These are mechanical-energy changes, not actual rocket fuel requirements.

19. Geostationary satellite

It appears stationary above one point on Earth. Required conditions:

- Circular orbit in the equatorial plane.
- Same direction as Earth's rotation.
- Period equal to Earth's rotation period: approximately 24 h (sidereal value about 86164 s).

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}, \qquad h = r - R.$$

For Earth, orbital radius is about 42200 km and altitude about 35800 km. Use the period and other constants supplied in the question. A geosynchronous satellite has the same period but need not stay fixed in the sky. A polar satellite orbits in a plane passing approximately through the poles; it is not geostationary.

20. Weightlessness

In an orbiting satellite, both the astronaut and the satellite are falling freely together. An ideal co-moving weighing scale reads zero; gravitational force is still present.

If a question includes planetary rotation

For a spherical planet, at the equator a body resting on the surface has

$$N = m(g_0 - \omega^2 R),$$

where N is the scale reading and ω is the planet's rotation rate. At the pole, $N = mg_0$. These formulas assume the body remains in contact with the surface and ignore oblateness.

Use with care. Altitude h is not orbital radius r. For geostationary calculations, find r first and subtract Earth's radius to get h.

Extension | Approximate effective gravity at latitude λ

$g_{\text{eff}}(\lambda) \simeq g_0 - \omega^2 R\cos^2\lambda$

For slow rotation of an ideal spherical planet: λ is latitude, ω is rotation rate; neglect oblateness and terms of order $(\omega^2 R/g_0)^2$.

08 — LAST-MINUTE REVISION

Ratios and common traps

Circular orbits around the same planet

Quantity	Changes with radius as	If radius becomes $4r$
Gravity g	$1/r^2$	$g/16$
Speed v_o	$1/\sqrt{r}$	$v_o/2$
Period T	$r^{3/2}$	$8T$
Kinetic energy K	$1/r$	$K/4$
Binding energy $\lvert E\rvert$	$1/r$	$\lvert E\rvert/4$
Angular momentum L	$\sqrt{r}$	$2L$

The energy and angular-momentum rows assume the **same satellite mass**. At the same radius around the same planet, all small satellites have the same circular speed and period, but their energies are proportional to mass.

Do not confuse these pairs

Quantity	Reminder
G and g	Universal constant versus local acceleration.
$R + h$ and h	Orbital radius versus altitude.
V and U	Potential per unit mass versus potential energy.
U and ΔU	Absolute value for a chosen reference versus change.
Gravity and apparent weight	A scale can read zero while gravity is non-zero.
Energy and binding energy	E is negative for a bound orbit; binding energy is its positive magnitude.

Six checks before substitution

1. Have I measured the distance from the centre?
2. Is the body outside, inside a solid sphere, or inside a shell?
3. Is uniform density or small height actually assumed?
4. Is the orbit circular or elliptical?
5. Is the object initially at rest or already orbiting?
6. Does the question ask for gravity's work or external work?

Use with care. Convert km to m and hours to seconds. Use the given constants. A higher circular orbit has lower speed but higher total energy; "higher energy" means less negative.

09 — SELF-CHECK

Six quick checks

Try mentally, then compare with the answers below. These are original checks, not claimed previous-year questions.

Questions

1. At height $h = R$ above a spherical planet, what is the gravity in terms of g_0?
2. At depth $d = R/4$ in a uniform planet, what is the gravity?
3. A body is moved slowly from exterior radius $2R$ to $4R$. Find the external work.
4. A body is launched vertically from the surface at half the surface escape speed. Find its maximum height above the surface.
5. A satellite's circular orbital radius increases from r to $4r$. What are the speed and period ratios?
6. A satellite of mass m is already in a circular orbit of radius r. How much extra energy is needed to escape?

Answers with one-line reasoning

1. $g_0/4$, because distance from the centre becomes $2R$.
2. $3g_0/4$, using $g_d = g_0(1 - d/R)$.
3. $+GMm/(4R)$, using $W_{\text{ext}} = \Delta U = GMm[1/(2R) - 1/(4R)]$.
4. $H = R/3$, using $u^2 = v_e^2/4$ in the maximum-height formula.
5. $v_2/v_1 = 1/2$ and $T_2/T_1 = 8$.
6. $GMm/(2r)$, because the initial total energy is $-GMm/(2r)$ and the final limiting energy is zero.

Revision advice. First learn the boxed formulas and their conditions. Then revise the graphs and ratio table. Use energy conservation when a direct shortcut is not obvious.