

Gravitation — Problem Set for Class Discussion

Questions for conceptual discussion and advanced problem-solving

PROBLEM SET DATE 12 October 2026	EXAM FOCUS JEE Advanced 2027	CHALLENGE 9 problems • 25 parts
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Designed for IIT-JEE 2027 Physics groups: advanced Gravitation practice that builds on JEE Main fundamentals and develops the multi-concept reasoning needed for JEE Advanced.

- Every part requires a labelled diagram or sketch where appropriate, and a step-by-step derivation.
 - Express final results symbolically before numerical substitution.
 - Use **G** for the gravitational constant. Unless stated otherwise, neglect air resistance, planetary rotation and other bodies.
 - Attempt each problem independently, then discuss the reasoning and alternative approaches in class.
- Difficulty: ★ standard; ★★ moderately hard; ★★★ hard.

Problem 01 — Radial fall into the Sun

Advanced • ★★

A probe of mass m is released from rest at Earth's orbital radius r_0 (1 AU). Earth's orbit is circular with period $T_0 = 365.25$ days. Neglect everything except the Sun's gravity (mass M).

- Using energy conservation, find the speed $v(r)$ of the probe when it is at distance r from the Sun.
- Treat the straight-line fall as half of a very thin ellipse of semi-major axis $r_0/2$ and use Kepler's third law to show that the time to reach the Sun is $T_0/(4\sqrt{2})$. Find this time in days.
- Find the time taken to cover the first half of the distance (from r_0 to $r_0/2$) as a fraction of the total fall time. (Hint: substitute $r = r_0\cos^2\theta$.)

Problem 02 — Ring and rod

Advanced • ★★

A uniform ring of mass M and radius R lies in a plane perpendicular to the x -axis with its centre at the origin. A thin uniform rod of mass m and length L lies along the x -axis from $x = d$ to $x = d + L$.

- Show that the gravitational field of the ring at a point on the axis at distance x from the centre is $GMx/(x^2 + R^2)^{3/2}$, directed towards the centre.
- Find the force of attraction between the ring and the rod.
- Find the force when the rod starts at the ring's centre ($d = 0$) and is very long ($L \rightarrow \infty$), with mass per unit length λ .

Problem 03 — Sphere with a cavity

Advanced · ★★

A uniform solid sphere of radius R has a spherical cavity of radius $R/2$ whose centre is at distance $R/2$ from the centre O of the sphere (so the cavity touches the surface and passes through O). Let M be the mass the sphere would have without the cavity.

- (a) Show that the gravitational field everywhere inside the cavity is uniform. Find its magnitude and direction.
- (b) Find the field at the surface point diametrically opposite to the cavity's outermost point (the point on the far side of O , on the line joining the centres).

Problem 04 — Planet with non-uniform density

Hard · ★★★

A planet of radius R and total mass M has density $\rho(r) = \rho_0(1 - r^2/R^2)$ for $0 \leq r \leq R$.

- (a) Express ρ_0 in terms of M and R . Find the mass $M(r)$ enclosed within radius r .
- (b) Find the total gravitational self-energy of the planet in the form $-\gamma GM^2/R$, and find γ . Compare it with the uniform-density value $3/5$ and explain the difference.
- (c) Find the radius at which the gravitational field $g(r)$ inside the planet is maximum, and the value of this maximum field in terms of GM/R^2 .

Problem 05 — Wire hanging in a borehole

Advanced · ★★

A narrow radial shaft is drilled from the surface of a non-rotating uniform planet (mass M , radius R). A uniform flexible wire of mass m , length L ($L \leq R$), cross-section A and Young's modulus Y hangs in the shaft from its top end at the surface. Inside the planet, $g(r) = g_0 r/R$ with $g_0 = GM/R^2$.

- (a) Find the tension $T(r)$ in the wire at distance r from the centre.
- (b) Find the maximum tension. Evaluate it for $L = R$.
- (c) Find the total elongation of the wire.

Problem 06 — Hohmann transfer

Advanced · ★★

A satellite is in a circular orbit of radius r with speed v_0 . Its engine fires briefly to place it on an elliptical orbit with perigee r and apogee $4r$. At apogee the engine fires again to make the orbit circular.

- (a) Find the speed at perigee just after the first burn and the increase in speed Δv_1 (in terms of v_0).
- (b) Find the speed at apogee before the second burn and the total Δv for both burns.
- (c) Find the time taken to go from perigee to apogee in terms of the period T_0 of the initial orbit.

Problem 07 — Unequal binary

Advanced · ★★

Two stars of masses M and $2M$, separated by a fixed distance D , move in circular orbits about their common centre of mass under mutual gravity.

- (a) Find the angular velocity and the orbital speed of each star.
- (b) Find the total kinetic energy, the total potential energy and the total mechanical energy of the system.

Problem 08 — Braking in orbit

Advanced · ★★

A satellite is in a circular orbit of radius r_0 and speed v_0 about a planet of radius R_p . A sudden burn reduces its speed to $f v_0$ ($0 < f < 1$) with the direction unchanged (still tangential).

- (a) Show that the point of burn becomes the apogee, and find the perigee distance and perigee speed for $f = 0.8$.
- (b) Find the period of the new orbit for $f = 0.8$ in terms of the original period T_0 .
- (c) If $R_p = r_0/2$, find the largest value of f for which the satellite will hit the planet.

Problem 09 — Chord tunnel through a uniform planet

Advanced · ★★

A smooth straight tunnel is drilled through a uniform planet (mass M , radius R) along a chord whose closest distance to the centre is d . A particle of mass m is released from rest at one end of the tunnel.

- (a) Show that the component of gravitational force along the tunnel is $-(GMm/R^3)s$, where s is the position measured along the tunnel from its midpoint. Hence show that the motion is simple harmonic and find its period. Does it depend on d ?
- (b) Find the maximum speed of the particle and the normal force exerted by the tunnel wall on it. Show that the normal force is constant.
- (c) Find the time taken to travel from one end of the tunnel to the other. Evaluate it for Earth ($R = 6400$ km, $g = 9.8$ m/s²), in minutes.

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